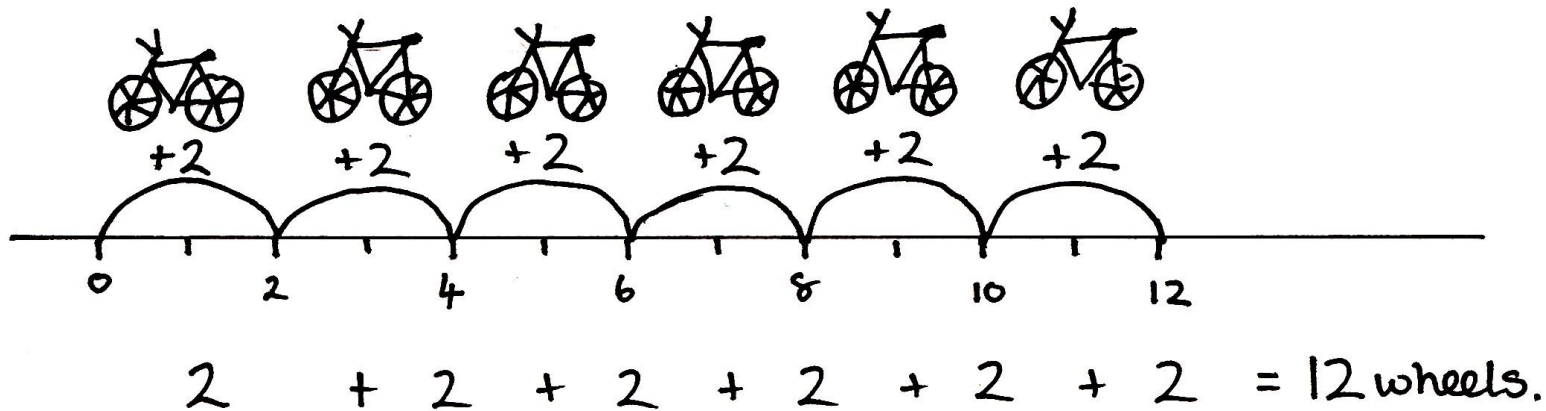


Progression in the use of the number line

Developing the number line as a visual image to support children's understanding of the number system and their strategies for calculation

Produced by The Hampshire Mathematics Advisory Team



Introduction

This booklet has been produced to provide practical examples from the Foundation Stage, through Year 1, Year 2 and into Year 3 to show how the image of a number line can support children's understanding in the following strands from the renewed framework:

- *using and applying mathematics*
- *counting and understanding number*
- *knowing and using number facts and calculating.*

The booklet is organised into sections for each year group. Each section identifies the appropriate learning objectives for the relevant strands of mathematics and then provides examples of the ways in which number lines could be used.

Since the early 1990s research has shown that the unstructured number line has been used successfully with pupils to develop mathematical understanding, (L Rousham, 2003). Children need regular opportunities to work with appropriate concrete number lines throughout the Foundation Stage, Key Stage 1 and Key Stage 2 in order to support their understanding of how the number system develops from whole numbers through to fractions and decimals. Number lines also support children's efficient recording of their mental strategies in all four operations and can help to develop children's understanding of the connections between them. They can be used to provide support for teacher assessment of children's thinking and children's own reflection on their understanding of mathematics.

The number line is a continuous line representing numbers to infinity. Children need to develop strategies where they are drawing a part of the number line to support their thinking for specific calculations. This image of the number line can help children to clarify the mental strategy used to solve a calculation. It is helpful to draw a line a little longer than is needed for the calculation to support this idea. The numbers on the unstructured number line should be labelled from left to right and written below the line. The strategy used to solve the calculation can then be clearly recorded above the line. Combining words and number sentences using symbols, alongside the recording of the number line helps children to make the links between mathematical recording and spoken calculations and discussion.

In order to use number lines effectively for addition and subtraction, children need to count on and back orally, in 1s, then in 10s, and then in 100s, from any given number. Particular emphasis needs to be given when crossing the 10s and then the 100s boundary. Equally, children often find counting back from a given number more challenging than counting on. It is important when adding or subtracting involving two-digit numbers to keep one number whole and partition the second number.

Initially children will be counting and recording in ones but gradually, with experience, they should be encouraged to calculate and record in 10s and 1s and then multiples of 10s and 1s and so on. They need to be encouraged to select the most efficient strategy to use the fewest steps they can when using number lines. An ongoing focus on children developing recall of key number facts to 10, 20 and 100 will also support children to select an efficient strategy for addition and subtraction.

Other models and images to support mathematical understanding of the number system and calculation can be found on the models and images posters and a number of associated Interactive Teaching Programmes (ITPs). These can be found in the resources folder of the Hampshire Mathematics Advisory Team's recent publication *Using the renewed Framework: Principles, key messages and guidance*. This CD-ROM has been sent to all schools, and is available on the Primary National Strategy website.

Reference: *The empty number line: A model in search of a learning trajectory?*, by L Rousham, in *Enhancing primary mathematics teaching*, edited by Ian Thompson, Publisher Open University Press 2003, ISBN: 0-335-21375-8. Original research by M Beishuizen, 1997. *Two types of mental arithmetic and the empty numberline*, paper presented at the British Society for Research in the Learning of Mathematics (BSRLM) conference, at Oxford University on 7 June 1997.

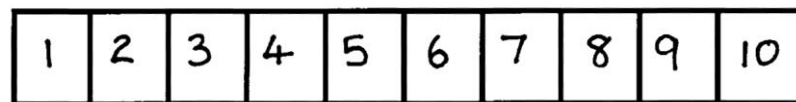
Learning objectives: *Foundation Stage, Year R*

- Use developing mathematical ideas and methods to solve practical problems.
- Match sets of objects to numerals that represent the number of objects.
- Sort objects, making choices and justifying decisions.
- Talk about, recognise and recreate simple patterns.
- Describe solutions to practical problems, drawing on experience, and talking about their own ideas, methods and choices.
- Say and use number names in order in familiar contexts.
- Know that numbers identify how many objects are in a set.
- Count reliably up to 10 everyday objects.
- Estimate how many objects they can see and check by counting.
- Count aloud in 1s, 2s, 5s or 10s.
- Use language such as *more* or *less* to compare two numbers.
- Use ordinal numbers in different contexts.
- Recognise numerals 1 to 9.
- Observe number relationships and patterns in the environment and use these to derive facts.
- Find one more or one less than a number from 1 to 10.
- Select two groups of objects to make a given total of objects.
- Begin to relate addition to combining two groups of objects and subtraction to *taking away*.
- In practical activities and discussion begin to use the vocabulary involved in adding and subtracting.
- Count repeated groups of the same size.
- Share objects into equal groups and count how many in each group.

Early skills to support visualisation and mental calculation

Use a number track from 1 to 10, and then 1 to 20, to support counting with objects and oral counting, set in meaningful contexts such as:

- *shall we count the wheels on the tricycles? Can you find that number on the number track?*
- *role play: How many plates do we need for the three bears? Can we put a circle around that number?*
- *if we stand on number 5 on the number track how many jumps must we make to reach the number 8?*
- *can you find the treasure hidden in the sand tray? Can you use the number track to find out how much treasure you have?*
- *there are five frogs sitting on the number track, one more jumps on. How many frogs are there now?*
- *throw the dice – can you show me that number on the number track. Can we count that number of pine cones?*
- *in the outside area there are five numbered parking spaces. Three cars are parked in the spaces. How many spaces are left?*



Learning objectives: Year 1

Using the number line to support visualisation and mental calculation

Using and applying mathematics

- Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to *pay* and *give change*.
- Describe a puzzle or problem using numbers, practical materials and diagrams, and use these to solve the problem and set the solution in the original context.
- Answer a question by selecting and using suitable equipment, and sorting information, shapes or objects. Display results using tables and pictures.
- Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions.
- Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures.

Counting and understanding number

- Count reliably at least 20 objects, recognising that when rearranged the number of objects stays the same.
- Compare and order numbers, using the related vocabulary.
- Read and write numerals from 0 to 20, then beyond. Use knowledge of place value to position these numbers on a number track and number line.
- Say the number that is one more or less than any given number, and 10 more or less for multiples of 10.

Knowing and using number facts

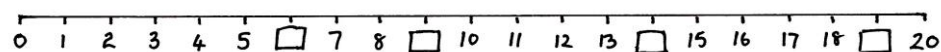
- Derive and recall all pairs of numbers with a total of 10.
- Count on or back in 1s, 2s, 5s and 10s.

Continue to use number tracks, but gradually introduce and make links with the number line, initially from 1 to 20, then 1 to 50.

Continue to use meaningful contexts to model counting and calculating using the number track and then the number line.

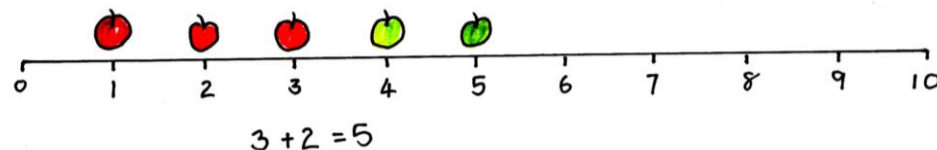
Using the number line to support counting and calculating

- Use the number line to develop a strong mental image of numbers when counting to 20, then beyond, forward and backwards in 1s.

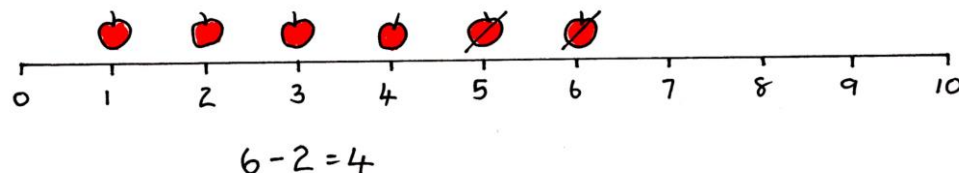


- Make links with the practical activity of counting how many objects there are altogether (addition), or taking away objects from a group (subtraction), to jumps on a number line.

If I have 3 apples and 2 apples, I have 5 apples altogether.



If I have 6 apples and I eat 2, I have 4 apples left.



Learning objectives: Year 1

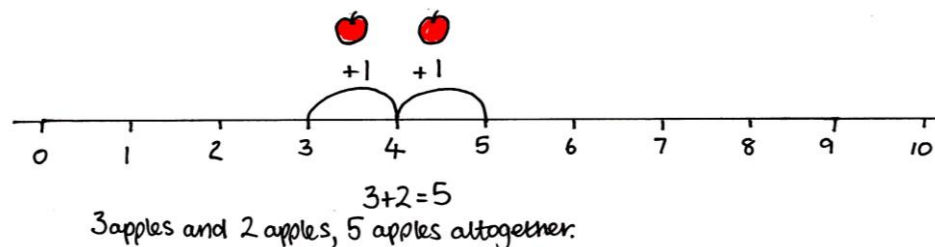
Using the number line to support visualisation and mental calculation

Calculating

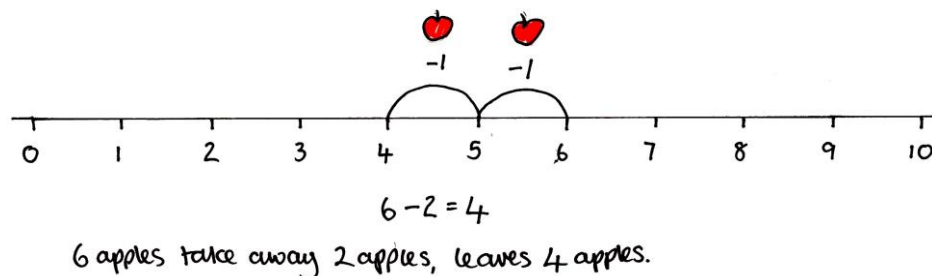
- Relate addition to counting on and recognise that addition can be done in any order. Use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number.
- Understand subtraction as *taking away* and counting back. Use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number.
- Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences.
- Solve practical problems that involve combining groups of two, five or ten.

- Link the practical activity of counting on or back to recording using a structured number line.

If I have 3 apples and 2 apples, I have 5 apples altogether.



If I have 6 apples and I eat 2, I have 4 apples left.



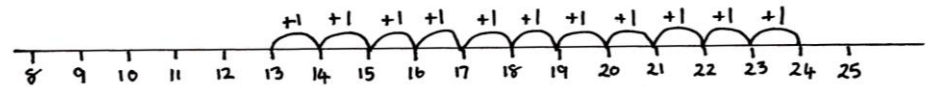
- Children begin to use the structured number line independently as a tool to support mental calculation. Alongside this they use number sentences to record the calculation.

Learning objectives: Year 1

Using the number line to support visualisation and mental calculation

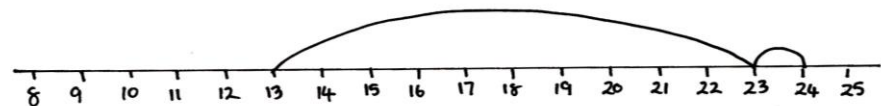
- Use the structured number line to support early place value knowledge, ie: addition of two *teen numbers* by partitioning one number and counting on in 10s and 1s.

If a farmer has 13 apples on one tree and 11 apples on another. How many apples does he have altogether?



13 apples and 11 apples
equals 24 apples.

$$13 + 11 = 24$$



13 apples and 10 apples,
and 1 apple, equals 24 apples.

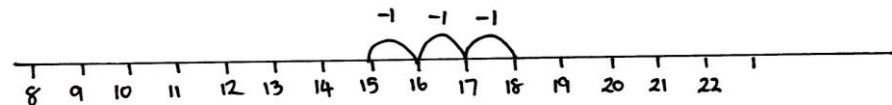
$$13 + 11 = 13 + 10 + 1$$

$$13 + 10 = 23$$

$$23 + 1 = 24$$

- Support subtraction of a single digit by counting back from a larger number.

If a farmer has 18 apples on a tree and he picks 3. How many apples will be left?



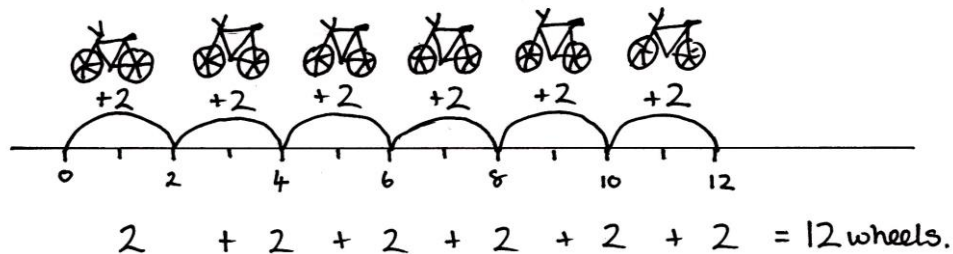
18 apples, take away 3 apples,
equals 15 apples.

$$18 - 3 = 15$$

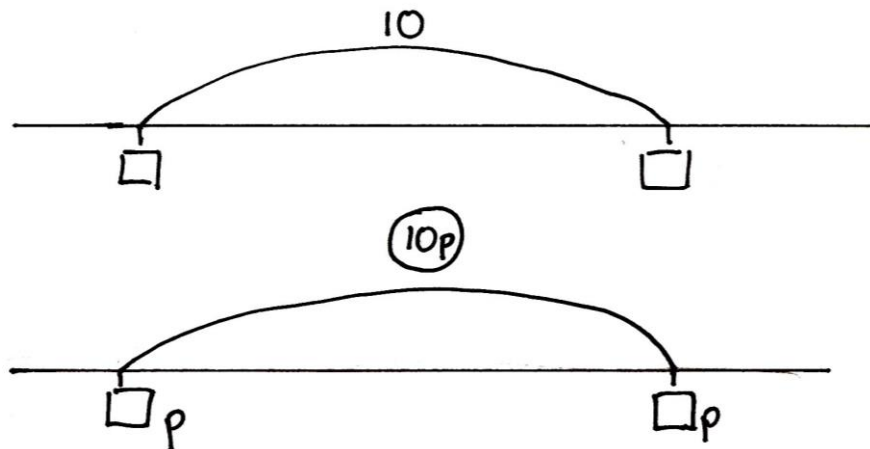
Using the number line to support other concepts

- Support counting forwards or backwards in 2s, 5s and 10s.

If I have 6 bicycles, how many wheels would there be?



What could the numbers be?

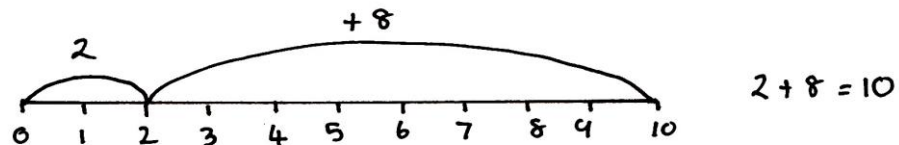
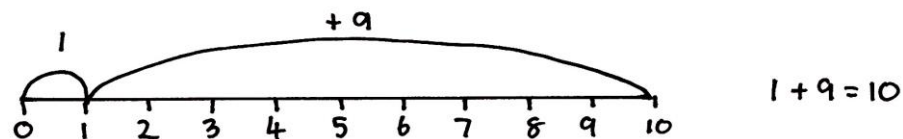


Learning objectives: Year 1

Using the number line to support visualisation and mental calculation

- Support understanding of number facts, ie: pairs of numbers that make 10.

A farmer has 10 apples in two baskets. If he has 1 in the first basket how many will be in the other basket? What if there was 2, 3, 4, 5, 6 ... in the first basket?



Learning objectives: Year 2

Using the number line to support visualisation and mental calculation

Using and applying

- Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures, or pounds and pence.
- Identify and record the information or calculation needed to solve a puzzle or problem. Carry out the steps or calculations and check the solution in the context of the problem.
- Follow a line of enquiry; answer questions by choosing and using suitable equipment, and selecting, organising and presenting information in lists, tables and simple diagrams.
- Describe patterns and relationships involving numbers or shapes, make predictions and test these with examples.
- Present solutions to puzzles and problems in an organised way. Explain decisions, methods and results in pictorial, spoken or written form, using mathematical language and number sentences.

Counting and understanding number

- Read and write two-digit and three-digit numbers in figures and words. Describe and extend number sequences and recognise odd and even numbers.
- Count up to 100 objects by grouping them and counting in 2s, 5s or 10s. Explain what each digit in a two-digit number represents, including numbers where 0 is a place holder. Partition two-digit numbers in different ways, including into multiples of 10 and 1.
- Order two-digit numbers and position them on a number line.
- Round two-digit numbers to the nearest 10.

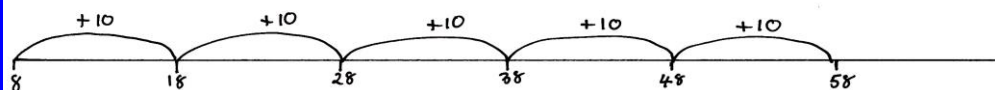
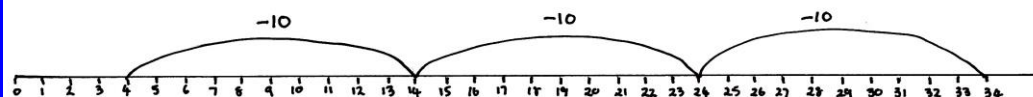
Children should have a secure mental image of the number system from 0 to 50 and begin to develop understanding of up to 100 and beyond.

The teacher should continue to model the use of a structured, and then an unstructured, number line to support counting and calculating, through meaningful contexts.

In Year 2 the number line should be used to support children's understanding of multiplication as repeated addition and division as grouping/repeated subtraction. Explicit links should constantly be made between the recording on the number line and the practical activity.

Using the number line to support counting and calculating

- Counting on and back in 1s, 10s and 100s from any two-digit number.
- Counting on and back in 2s and 5s from 0.
- Explore patterns using jumps of a constant size, forwards and backwards, starting from any number.



Learning objectives: Year 2

Using the number line to support visualisation and mental calculation

Knowing and using number facts

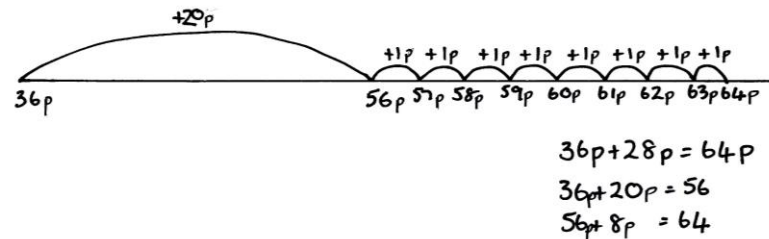
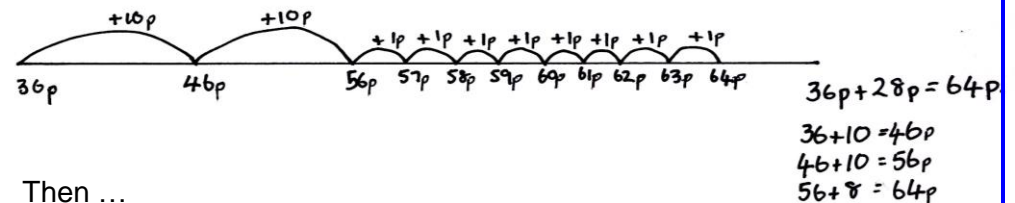
- Derive and recall all addition and subtraction facts for each number to at least 10, all pairs with totals to 20 and all pairs of multiples of 10 with totals up to 100.
- Understand that halving is the inverse of doubling and derive and recall doubles of all numbers to 20, and the corresponding halves.

Calculating

- Add or subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number. Use practical and informal written methods to add and subtract two-digit numbers.
- Understand that subtraction is the inverse of addition and vice versa. Use this to derive and record related addition and subtraction number sentences.
- Use the symbols $+$, $-$, \times , \div and $=$ to record and interpret number sentences involving all four operations. Calculate the value of an unknown in a number sentence.

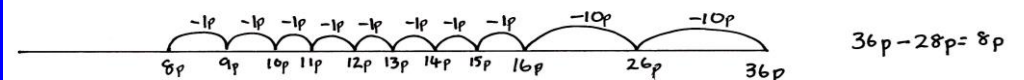
- Adding two, two-digit numbers by partitioning one number and counting on in 10s and 1s then multiples of 10 and 1.

I have 36p and my mum gives me 28p pocket money. How much money do I have altogether?

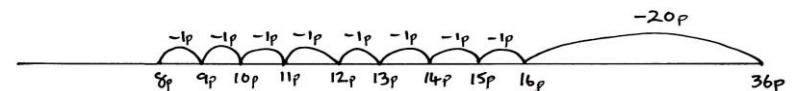


- Subtracting two, two-digit numbers by partitioning the second number and counting back in 10s and 1s then multiples of 10 and 1.

I have 36p, I spend 28p. How much do I have left?



Then ...



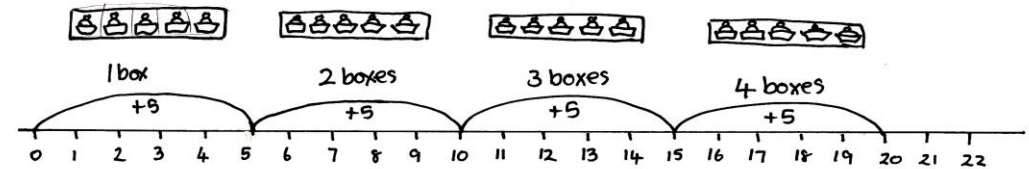
Learning objectives: Year 2

- Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division. Use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders.

Using the number line to support visualisation and mental calculation

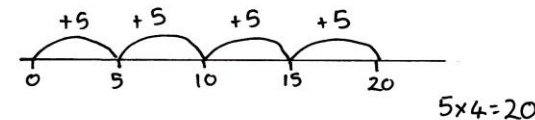
- Multiplying through making links to counting on in steps of equal size. (multiples of 2s, 5s and 10s, then other numbers).

There are 5 cakes in one box. How many cakes in 4 boxes?



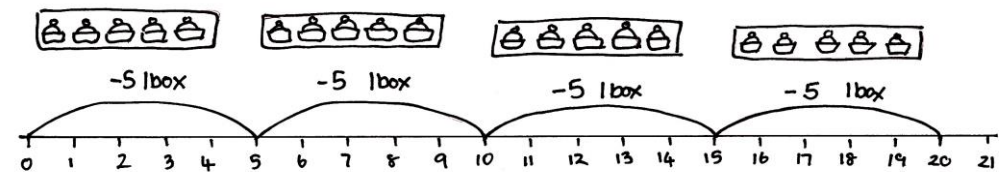
$$5 \times 4 = 20$$

5 cakes in each box. 4 boxes equals 20 cakes altogether.



- Dividing through making links to counting back in steps of equal size. (multiples of 2s, 5s and 10s).

I have 20 cakes, I can fit 5 cakes in a box. How many boxes will I need?



*20 cakes divided into boxes of 5,
I need 4 boxes.*

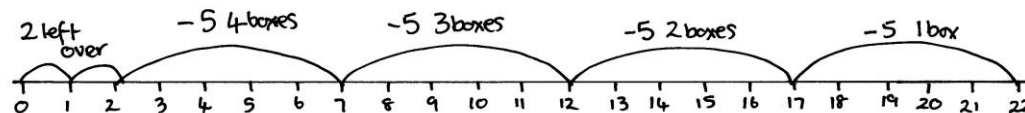
$$20 \div 5 = 4 \text{ boxes}$$

There are 4 groups of 5 in 20.

Learning objectives: Year 2

Using the number line to support visualisation and mental calculation

What if I had 22 cakes to pack?

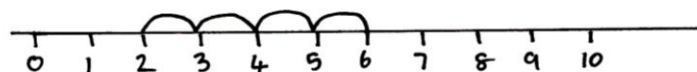


$$22 \div 5 = 4 \text{ boxes, with } 2 \text{ cakes left over.}$$

Using the number line to support other concepts

- Explore the concept of inverses.

What might the calculation be?



$$\begin{aligned} 2 + 4 &= 6 \\ 6 - 4 &= 2 \end{aligned}$$

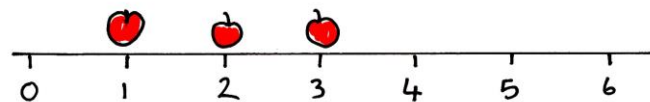


$$\begin{aligned} \square + 4 &= \square \\ \square - 4 &= \square \end{aligned}$$

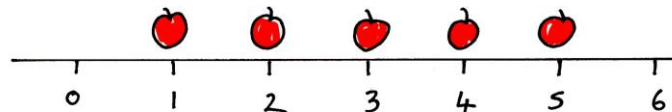
Learning objectives: Year 2

Using the number line to support visualisation and mental calculation

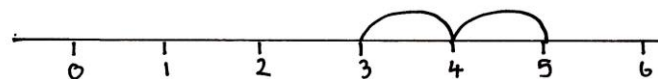
- Support the concept of the difference between two numbers.



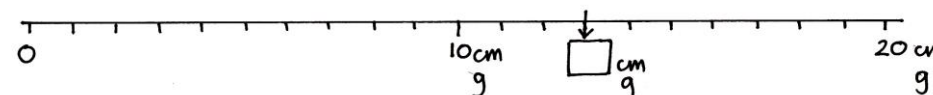
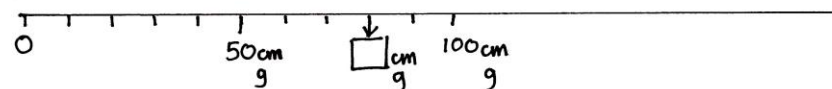
Sam has 3 apples.
Anne has 5 apples.
How many more apples
does Anne have?



What is the difference
between 3 and 5?



- Support reading the numbered divisions on a scale, and interpreting the divisions between them.



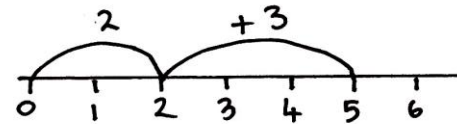
Measuring

Read the numbered divisions on a scale, and interpret the divisions between them (eg: on a scale from 0 to 25 with intervals of 1 shown but only the divisions 0, 5, 10, 15 and 20 numbered). Use a ruler to draw and measure lines to the nearest centimetre.

Learning objectives: Year 2

Using the number line to support visualisation and mental calculation

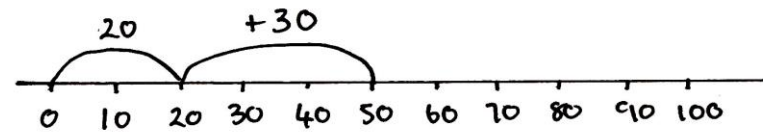
- Explore patterns in calculation, ie: pairs of multiples of 10 with totals up to 100.



$$2+3=5$$

Therefore ...

$$20+30=50$$



Learning objectives: Year 3

Using the number line to support visualisation and mental strategy

Using and applying mathematics

- Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations.
- Represent the information in a puzzle or problem using numbers, images or diagrams, and use these to find a solution and present it in context, where appropriate using £.p notation or units of measure.
- Identify patterns and relationships involving numbers or shapes, and use these to solve problems.
- Describe and explain methods, choices and solutions to puzzles and problems, orally and in writing, using pictures and diagrams.

Counting and understanding number

- Read, write and order whole numbers to at least 1,000 and position them on a number line. Count on from and back to zero in single-digit steps or multiples of 10.
- Round two-digit or three-digit numbers to the nearest 10 or 100.

Knowing and using number facts

- Derive and recall all addition and subtraction facts for each number to 20, sums and differences of multiples of 10, and number pairs that total 100.
- Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and the corresponding division facts. Recognise multiples of 2, 5 or 10 up to 1,000.
- Use knowledge of number operations and corresponding inverses.

Children should have a secure mental image of the number system up to 1,000 and beyond.

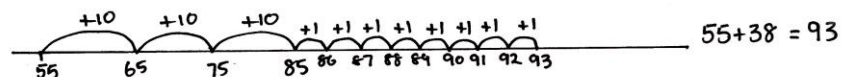
The teacher should continue to model the use of a structured or an unstructured number line to support counting and calculating, through meaningful contexts.

Building on from previous experience, in Year 3 the number line should be used to support children's growing efficiency in calculating.

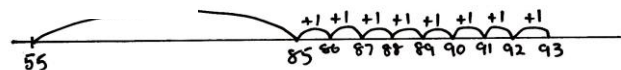
Using the number line to support counting and calculating

- Crossing the 10s boundary by either, counting in 1s, counting in 10s and 1s or partitioning the final digit.

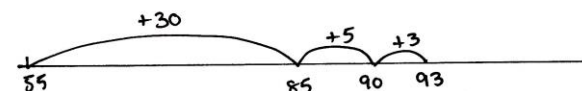
There are 38 fiction books and 55 non-fiction books in the library. How many books are there in the library?



Then ...



Then ...



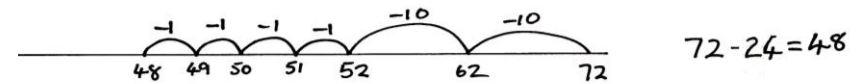
Learning objectives: Year 3

Using the number line to support visualisation and mental strategy

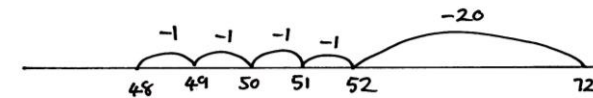
Calculating

- Add or subtract mentally combinations of one-digit and two-digit numbers.
- Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers.
- Understand that division is the inverse of multiplication and vice versa. Use this to derive and record related multiplication and division number sentences.

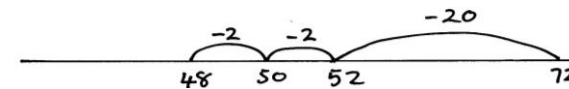
There were 72 books in the library. Children borrowed 24 of them. How many books were left in the library?



Then ...

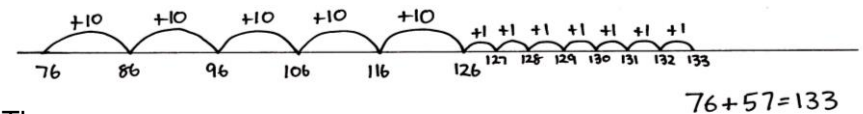


Then ...

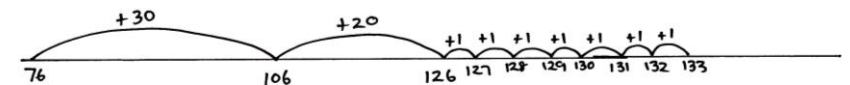


- Crossing the 100s boundary.

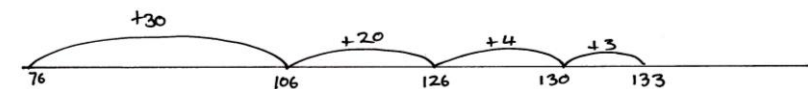
There are 76 marbles in one jar and 57 marbles in another jar. How many marbles are there altogether?



Then ...



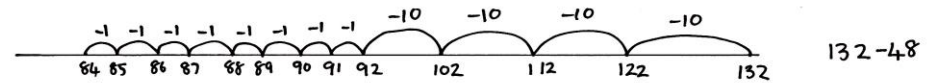
Then ...



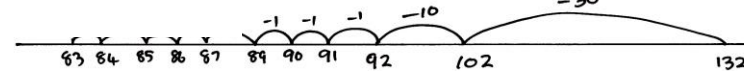
Learning objectives: Year 3

Using the number line to support visualisation and mental strategy

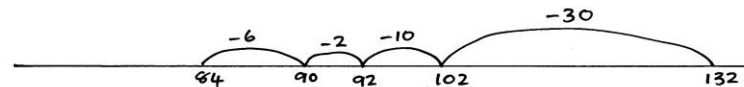
There were 132 marbles in a jar. Sarah took 48 out of the jar. How many marbles were left?



Then ...



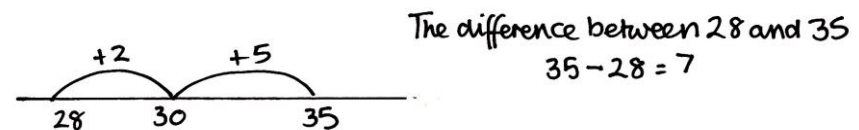
Then ...



Subtraction as difference ...

When children understand the concept of difference, through practical activity and can confidently subtract by counting backwards, they are ready to begin to use *counting on* to find the difference *if the numbers are close together*, eg: difference between 28 and 35. Some more-able children may be ready to use this strategy much earlier. Ideally children should be encouraged to look at the numbers in a calculation and decide for themselves whether it is better to count on, or to count back.

Sam has 28 pencils and Sarah has 35 pencils. How many more pencils does Sam need to have the same number of pencils as Sarah?



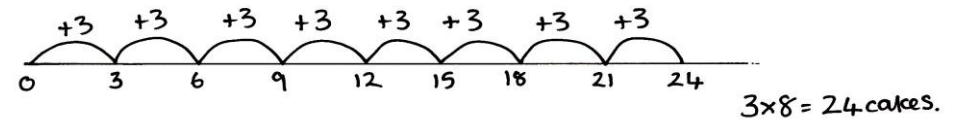
Learning objectives: Year 3

- Multiply one-digit and two-digit numbers by 10 or 100, and describe the effect.
- Use practical and informal written methods to multiply and divide two-digit numbers, eg: 13×3 , $50 \div 4$. Round remainders up or down, depending on the context.

Using the number line to support visualisation and mental strategy

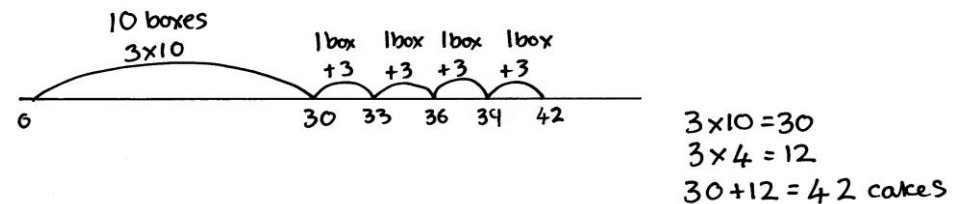
- Multiplying through making links to counting on in steps of equal size (multiples of 2s, 3, 4, 5, 6s and 10s, then other numbers).

There are 3 cakes in one box. How many cakes in 8 boxes?



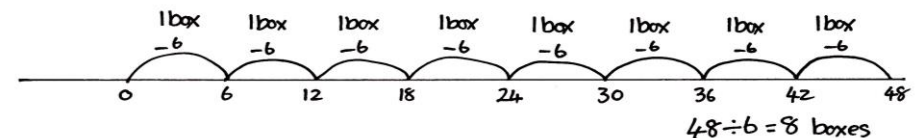
- Using known facts to multiply a *teens number* by a single digit.

There are 3 cakes in one box. How many cakes in 14 boxes?



- Dividing through making links to counting back in steps of equal size (multiples of 2, 3, 4, 5, 6s and 10s).

I have 48 cakes, I can fit 6 cakes in a box. How many boxes will I need?



Learning objectives: Year 3

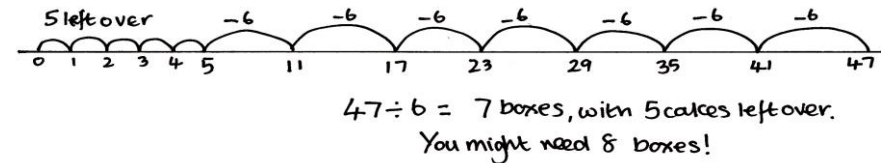
Using the number line to support visualisation and mental strategy

- Understand that some division calculations have remainders and make a decision about rounding up or down according to the context.

I have 47 cakes. I can fit 6 cakes in a box. How many boxes will I need?

or

I have 47p and lollipops cost 6p. How many lollipops could I buy? Would I have any money left over?

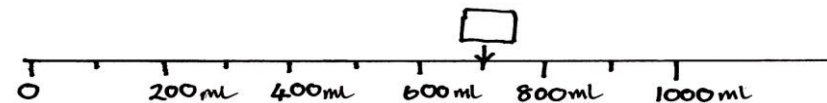


Measuring

- Read, to the nearest division and half-division, scales that are numbered or partially numbered. Use the information to measure and draw to a suitable degree of accuracy.
- Read the time on the 12-hour digital clock and to the nearest five minutes on an analogue clock. Calculate time intervals and find start or end times for a given time interval.

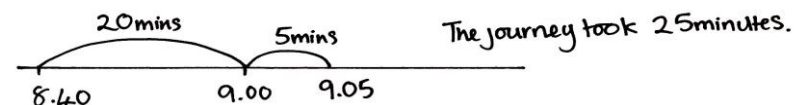
Using the number line to support other concepts

- Support reading, to the nearest division and half-division, scales that are numbered or partially numbered.



- Support calculating time intervals and find start or end times for a given time interval.

If Mary leaves home at 8.40 am and arrives at school at 9.05 am, how long did her journey take?

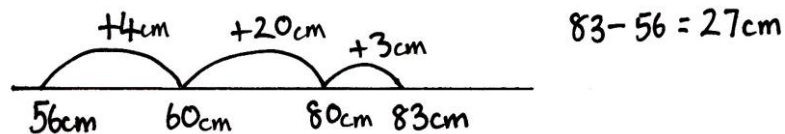


Learning objectives: Year 3

Using the number line to support visualisation and mental strategy

- Support calculation in the context of money and measures.

Paul has two snakes, one is 56cm long and the other snake is 83cm long. What is the difference in their length?



Molly buys a comic for £1.37 and a book for £2.56. How much does she spend altogether?

